ANISOTROPIC AND NONLINEAR PROPERTIES OF ROCK INCLUDING FLUID UNDER PRESSURE

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Abstract

This paper presents the mathematics and procedures to determine elastic rock properties from testing cylindrical core based upon orthotropic elastic theory. It also examines the extremely non-linear, but elastic, stress strain characteristics of some sandstones and what controls their Young's moduli and Poisson's ratios. The effects of fluid pressure changes within the rock are also considered. Two different modes of fluid behaviour are considered. The first is associated with poroelastic behaviour, while the second is associated with the effect of fluid within fractures. The use of these parameters leads to stress distributions and deformations that vary from those arrived at using conventional, but incorrect, assumptions of rock behaviour.

Keywords: *rock, effective stress, Young's moduli, fluid pressure, rock, poroelastic*

Introduction

Fluids may be thought to act mechanically within rock in two different ways. The first is by a poroelastic response to fluid pressure which affects the deformation of the rock mass. The second is by the direct action of the fluid within fractures. In each case the fluid pressure changes what may be considered to be the effective stress within the rock, but in different ways.

If we examine the general equation for effective stress within a rock mass it may be thought to follow the form of Equation 1 (Gray, 2017).

$$\sigma'_{ij} = \sigma_{ij} - \delta_{ij}\alpha_i P \tag{1}$$

where: σ'_{ij} is the effective stress on a plane perpendicular to the vector i in the direction j.

σ_{ij} is the total stress on a plane perpendicular to the vector i in the direction j.

δ_{ij} is the Kronecker delta. If $i \neq j$ then $\delta_{ij} = 0$, while if $i = j$ then $\delta_{ij} = 1$.

α_i is a poroelastic coefficient affecting the plane perpendicular to the vector i. Its value lies between 0 and 1.

P is the fluid pressure in pores and fractures within the rock.

The Kronecker delta term is used because a static fluid cannot transmit shear.
The directional subscript indicating direction in the poroelastic coefficient is not usual practice where, for measurement reasons, only a scalar value is obtained.

If the choice of coordinates aligns with that of an open joint, then the poroelastic coefficient orthogonal to the joint is unity. More generally in a porous rock mass it lies somewhere between zero and unity. In a volcanic glass the values of the poroelastic coefficients are zero.

Deformation and the Determination of the Stiffness Matrix from Rock Core

The deformation of a rock mass is dependent upon the stresses it is subject to and its stiffness. If a rock is subject to a complex stress with six components of direct and shear stress it will deform to produce six strains. The correlation between stress and strain is a compliance matrix of 36 components. Determining all of these is practically impossible. The common simplification where the rock is treated as being isotropic, having a single value of Young's modulus and Poisson's ratio, is however frequently in error.

If we make a simplifying assumption that the rock mass is orthotropic then the compliance matrix has twelve components. These are shown in Equation 2 below.

$$\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_1} & -\frac{v_{21}}{E_2} & -\frac{v_{31}}{E_3} & 0 & 0 & 0 \\ -\frac{v_{12}}{E_1} & \frac{1}{E_2} & -\frac{v_{32}}{E_3} & 0 & 0 & 0 \\ -\frac{v_{13}}{E_1} & -\frac{v_{23}}{E_2} & \frac{1}{E_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{23}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{31}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{Bmatrix} \tag{2}$$

If we take a sample that is aligned perpendicularly to some plane of obvious symmetry then we have reduced the unknowns. An example of this is a core drilled perpendicularly to the bedding planes of a sedimentary rock. If we subject that core to testing in a triaxial loading rig which can apply a stress along the axis of symmetry of a round core sample and a confining stress perpendicular to that axis, we can measure the resulting axial and perpendicular strains with each stress increment. If there are axial strain gauges and at least three tangential strain gauges it is possible to calculate the major and minor tangential strains in addition to the axial strain. We can therefore derive three strains which can be assumed to represent the orthogonal cases.

Because we are dealing with principal stresses and strains the compliance matrix to be solved for the rock behaviour has therefore been reduced to nine components. It may be reduced still further to six unknowns because the matrix is symmetrical. This means that the off diagonal components are equal as shown in Equation 3. This symmetry provides a link between the values of Young's moduli and Poisson's ratios.

$$\frac{v_{ij}}{E_i} = \frac{v_{ji}}{E_j} \tag{3}$$

However three principal strains and two loading cases does not provide an adequate basis to determine the six unknowns. To determine the three Young's moduli and associated Poisson's ratios it is necessary to assume something. The assumption that we have made to determine the values of Young's moduli and Poisson's ratios is shown in Equation 4. Here the geometric mean Poisson's ratio, v_a, is assumed to have the same value for the three combinations of v_{ji}.

$$\sqrt{(v_{ij} v_{ji})} = v_a \tag{4}$$

In a triaxial situation the axial load, in the 1 axis, may be changed and the associated strain changes measured. From this loading and strain measurement the axial Young's modulus, E_1, and the two Poisson's ratios, v_{12} and $v_{13,}$ may be directly determined. However when the confining stress is changed, the situation is more complex to analyse because the 2 and 3 axis loadings are the same and applied simultaneously. Equation 5 describes strain in an elastic solid under three varying stresses.

$$\Delta\varepsilon_i = \frac{1}{E_i}\Delta\sigma_i - \frac{v_{ji}}{E_j}\Delta\sigma_j - \frac{v_{ki}}{E_k}\Delta\sigma_k \tag{5}$$

Using the relationship of Equation 3, Equation 5 may be re-written as Equation 6.

$$\Delta\varepsilon_i = \frac{1}{E_i}\left(\Delta\sigma_i - v_{ij}\Delta\sigma_j - v_{ik}\Delta\sigma_k\right) \quad (6)$$

Equation 6 can in turn be re-written using the relation of Equation 4 as Equation 7.

$$E_i = \frac{1}{\Delta\varepsilon_i}\left(\Delta\sigma_i - \sqrt{\frac{E_i}{E_j}}v_a\Delta\sigma_j - \sqrt{\frac{E_i}{E_k}}v_a\Delta\sigma_k\right) \quad (7)$$

Three nonlinear equations of the form of Equation 7 may be derived for each of the principal Young's moduli. These may be solved simultaneously using some value of the geometric mean Poisson's ratio, v_a. A specific value of v_a will solve Equation 7 to provide the same value of E_1 as derived from a purely axial stress change at the same stress range. This value of v_a provides a basis for the determination of the other values of Young's moduli and Poisson's ratios. A series of step changes in axial and confining loading can thus be used to determine the orthotropic moduli of a core sample.

Poroelastic Effects

If we now consider the case where the rock contains fluid in its internal pore space Equation 5 may be rewritten in terms of effective stress as causing deformation simply by replacing $\Delta\sigma_i$ with $\Delta\sigma'_i$. By substituting Equation (1) into Equation (5) for effective stress, we may arrive at Equation (8) which describes deformation in terms of the three principal total stresses, fluid pressure and the poroelastic coefficients applying to these directions.

$$\Delta\varepsilon_i = \frac{1}{E_i}\Delta\sigma_i - \frac{v_{ji}}{E_j}\Delta\sigma_j - \frac{v_{ki}}{E_k}\Delta\sigma_k - \Delta P\left(\frac{1}{E_i}\alpha_i - \frac{v_{ji}}{E_j}\alpha_j - \frac{v_{ki}}{E_k}\alpha_k\right) \quad (8)$$

As described previously it is possible to determine the orthotropic Young's moduli and Poisson's ratio for a core sample by a stepwise testing process involving changes in axial and confining stress. If we also incorporate cycles of fluid pressure variation into the test routine on the same strain gauged sample, it is possible to determine the values of the poroelastic coefficients via simultaneous solution of the three strain equations based on Equation 8 (Gray, 2017). The poroelastic coefficients so determined are in the direction of the axes of the principal orthogonal stiffness determined by the test procedure previously described. It is possible that the principal directions of the poroelastic coefficients are in fact different from the elastic ones. Unlike the work by Biot and Wills (1957) the poroelastic coefficients derived are a tensor. Their results are a function of the volumetric determination of poroelastic behaviour.

Experimental Results

Let us examine the results of testing and analysis of Hawkesbury sandstones from the Sydney area of New South Wales, Australia. These samples have been core drilled approximately perpendicular to their bedding planes. The core diameter was 61 mm.

The first sample is of a porous medium grained sandstone. Figure 1 shows its Young's modulus perpendicular to the bedding plane plotted against axial (perpendicular to the bedding plane) and confining stress. Figure 2 shows the Young's modulus in a direction parallel to the bedding plane. In each case Young's modulus increases with stress and is primarily a function of the stress in the same direction.

Figure 3 shows the Poisson's ratio associated with axial (cross bedding) stress and deformation parallel to the bedding plane. It characteristically shows an increase in Poisson's ratio with shear stress. Figure 4 shows the poroelastic coefficient in the direction of the axis of the sample. It also increases with shear stress. Both the increase in Poisson's ratio and poroelastic coefficient may be dependent on some level of dilation.

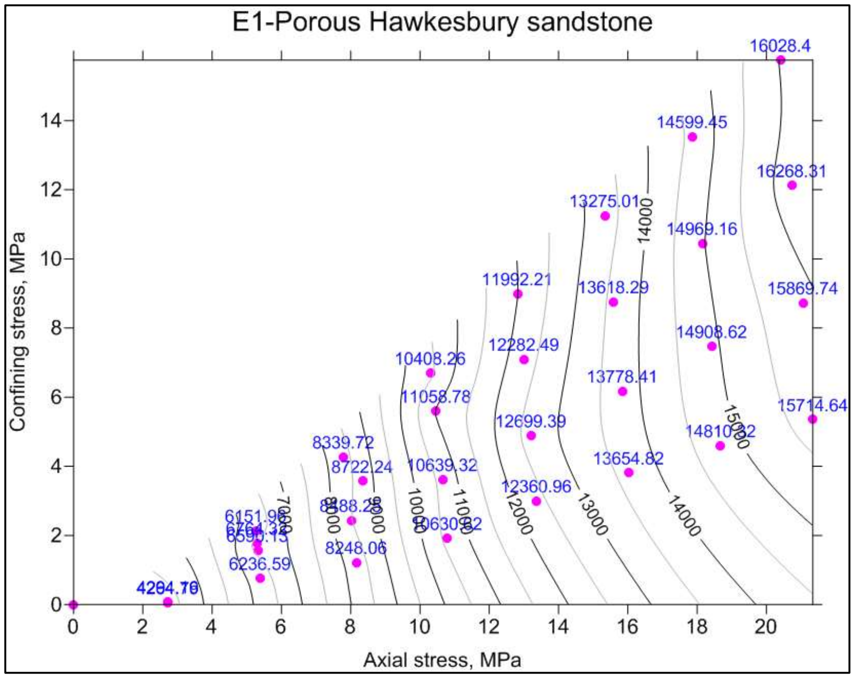

Figure 1. The Young's modulus perpendicular to bedding of a porous sandstone (MPa).

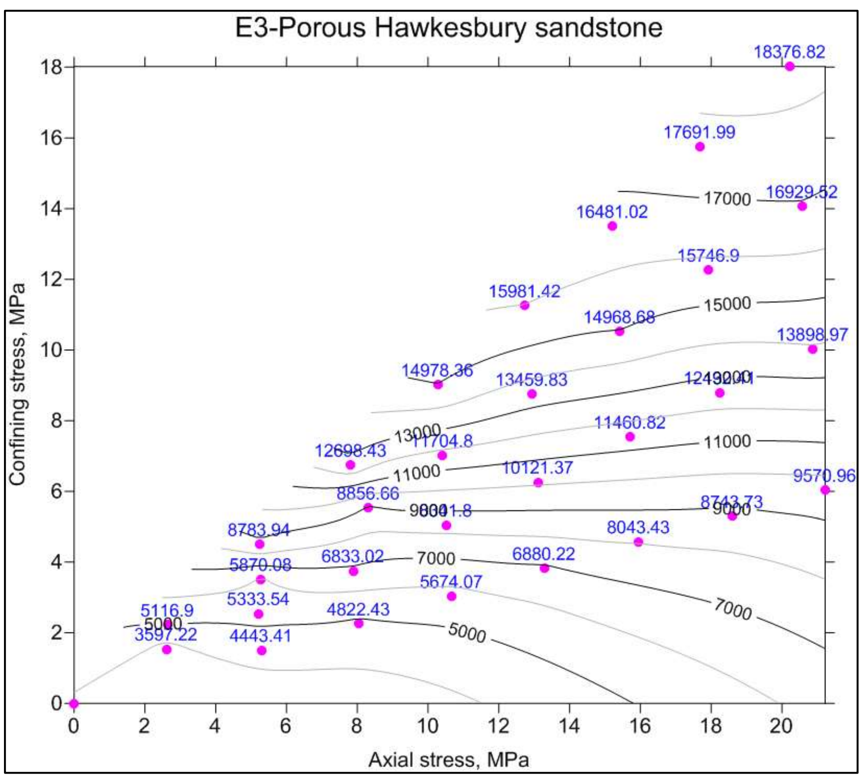

Figure 2. The Young's modulus parallel to bedding of a porous sandstone (MPa).

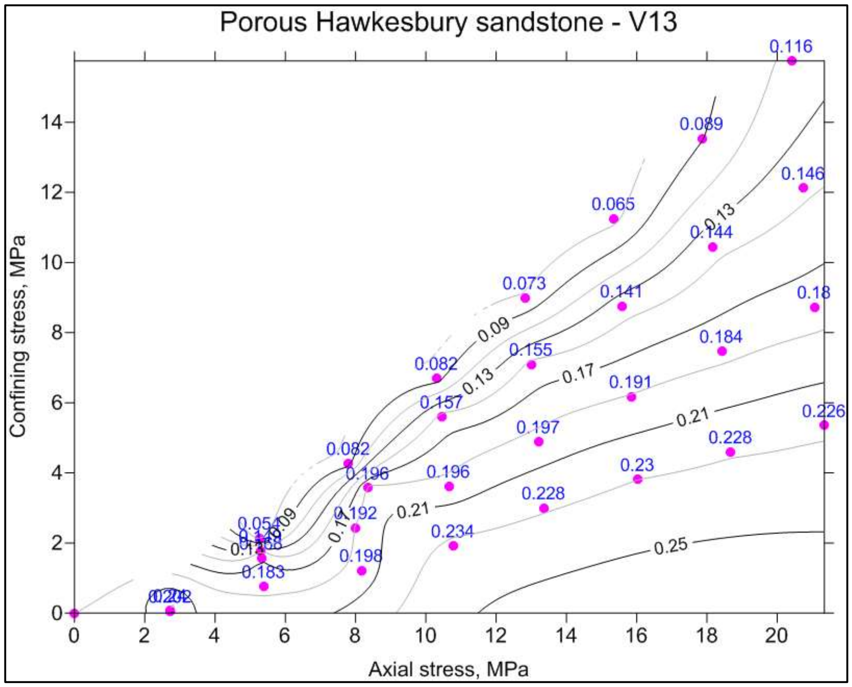

Figure 3. The Poisson's ratio associated with cross bedding stress and deformation parallel to the bedding for a porous sandstone.

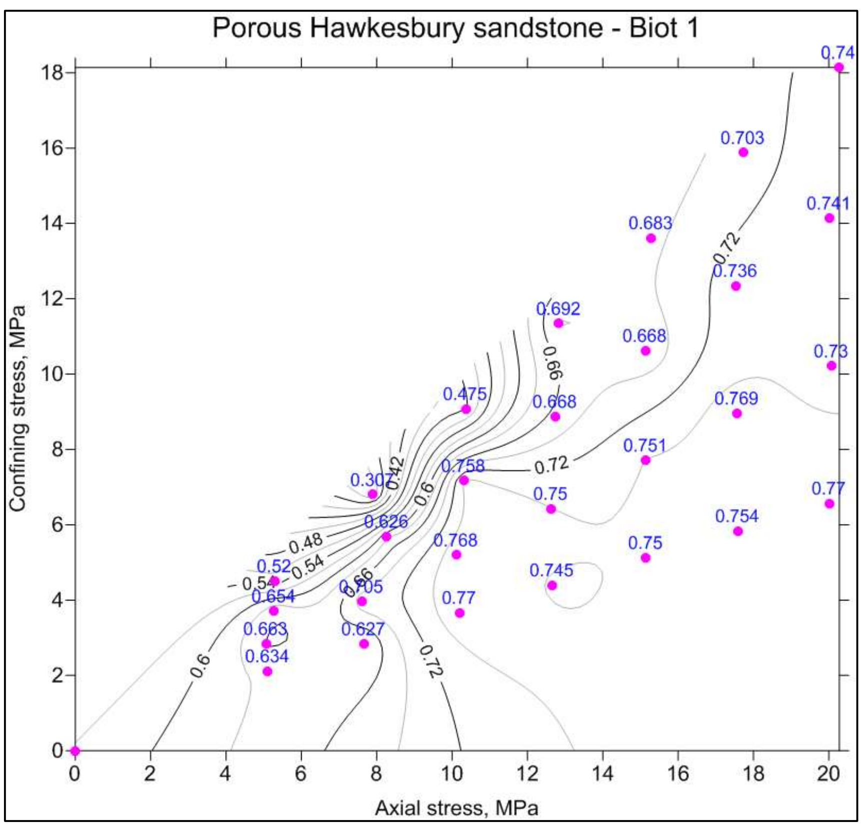

Figure 4. The poroelastic coefficient referred to the cross bedding axis of a porous sandstone.

Testing of a fine to medium grained low porosity sample of Hawkesbury sandstone containing some silty and clay components provided some quite different characteristics. The value of the Young's modulus across the bedding plane (E_1) was dependent on both the axial and confining stresses and the poroelastic coefficients were very small.

Testing on granite from the Snowy Mountains area of New South Wales has shown generally isotropic behaviour with a virtually constant Young's modulus and Poisson's ratio and a negligible poroelastic coefficient. The poroelastic coefficient appears sensible given the crystalline nature of the rock, but is at variance with values reported by Detournay and Cheng, 1993.

Fractured Rock Behaviour

The previous sections deal with poroelastic behaviour. The effective stress associated with this is an apparent effective stress dependent on the action of fluid within the pores, and micro fractures of the rock causing strain within the rock matrix. If the rock contains clear fractures then the term α_i describes a ratio of fracture area to total area over which fluid acts. This case has no relation to the poroelastic behaviour of the rock matrix.

Conclusions

This paper describes the process and mathematics involved in obtaining orthotropic elastic parameters including poroelastic behaviour from the triaxial testing of core. To do this requires the assumptions that the rock behaves in an orthotropic manner, that a likely axis of symmetry is common with the core axis and that a unique value of a geometric mean Poisson's ratio exists for each stress state.

Using the procedures and mathematics outlined the results of testing Hawkesbury sandstones show that the Young's moduli are highly dependent on the state of stress, varying some four to five fold from zero to 20 MPa axial and confining load. In a porous sample the Young's moduli seem to be dependent on the state of stress in the direction being measured. In the case of a low porosity sample the Young's moduli were dependent on both the axial and confining stress. In this sample the poroelastic coefficients were close to zero. Tests on some siltstones have shown less variation of modulus with stress, however the general trend of increasing modulus with stress exists. Some coals tested show an increase in stiffness of an order of magnitude. Generally we have found that sedimentary rocks we have tested show an anisotropy of less than 1.5:1 but with some exceptions which are nearly 5:1.

Most rock mechanics design is based on linear elastic models until strength based failure is reached. In addition the effects of fluid pressure are generally ignored. This paper shows that these assumptions are, in the case of the sedimentary rocks tested, quite incorrect. The elastic but very nonlinear behaviour is of particular importance. The consequences are that predicted deformations and stresses using linear elastic assumptions will be, in some cases, quite significantly in error.

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