

# The Fundamentals of Permeability Testing

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**ABSTRACT:** This paper examines the practice of permeability testing and demonstrates how many of the current test processes used by many geotechnical engineers to produce results that are seriously in error; prime culprits are packer and slug tests. It then presents the alternatives that can be used to obtain values of permeability that have some meaning. The key to producing sensible results is the correct analysis of the transient pressure response to flow, including methods to eliminate changes in permeability around the test hole. The use of single and multiple hole tests is considered. In the case of the latter, the use of pulse testing to obtain directional permeability and storage parameters is also presented. The work follows current trends in petroleum well testing but adapted for geotechnical engineering.

**KEYWORDS:** Permeability, hydraulic conductivity, fluid pressure, field testing.

## 1 INTRODUCTION

Permeability relates the velocity of fluid flowing in the ground under a laminar flow regime to the potential gradient. The potential in this case has both pressure and gravitational components. This is described in equation 1.

$$u = -\frac{k}{\mu}(\nabla p - \gamma \nabla z) \quad (1)$$

Where  $u$  is the apparent flow velocity (a tensor)  
 $k$  is the permeability  
 $\nabla p$  is the pressure gradient  
 $\gamma$  is the density of the fluid  
 $\nabla z$  is the height gradient  
 $\mu$  is the dynamic viscosity of the fluid

In groundwater applications Equation 1 is generally simplified to Equation 2.

$$u = -K(\nabla h) \quad (2)$$

Where  $K$  is the hydraulic conductivity  
 $\nabla h$  is the head gradient

In both cases  $k$  or  $K$  is a second order tensor with diagonal terms only. The keys to flow are the potential gradient, the permeability and the viscosity of the fluid. The head may be measured as the height to which a column of water would rise in a tube from the point of measurement.

From a geotechnical viewpoint, of key importance is the pressure of the fluid, as it is a component of effective stress, and the rate of fluid flow. The latter may matter from the viewpoint of sizing dewatering equipment or water supply. What is often more important though, is the variation in permeability between different parts of the ground. The reason for this is that this distorts the pressure distribution from one that might be expected in homogeneous ground.

For an understanding of steady state flow in saturated ground, all that is required is the measurement of permeability and pressure (head) and its variation. If an understanding of the transient response of flow in the ground is required then it is necessary to understand storage behaviour. This is the amount of fluid it will take per unit volume for a change in pressure. In parts of the ground that may make a transition between being partially and fully saturated, the nature of storage changes from one of changing porosity with pressure to the partial filling and emptying of pore space. There are additional complications associated with flow in unsaturated porous media, such as the effects of relative permeability and capillary pressure.

## 2 THE ANALYTICAL BASIS OF MEASUREMENT

Virtually all permeability measurement used in geotechnical engineering is conducted down a borehole. An exception to this is vertical infiltration tests, which involve pouring water on to the ground surface.

Flow from a borehole is assumed from an analytical viewpoint to be radial. It is therefore governed by Equations 4 and 5 which describe the pressure changes with respect to radial distance and time after the start of flow.

$$p_{r,t} = p_i - \frac{q\mu}{4\pi kh} \int_x^\infty \frac{e^{-s}}{s} ds \quad (4)$$

$$x = \frac{\phi\mu cr^2}{4kt} \quad (5)$$

Where  $p_{r,t}$  is the pressure  
 $p_i$  is the initial pressure  
 $q$  is the flow rate  
 $h$  is the test zone thickness  
 $\phi$  is the porosity  
 $c$  is the total compressibility  
 $r$  is the radius from the well  
 $t$  is the time of flow  
 $\int_x^\infty \frac{e^{-s}}{s} ds$  is the exponential integral

Where the value of Equation 5 is less than 0.01 then equation 4 may be take the simpler form of Equation 6.

$$p_{r,t} = p_i - \frac{q\mu}{4\pi kh} \ln \frac{4kt}{\gamma\phi\mu cr^2} \quad (6)$$

Where  $\gamma = 1.718$

For virtually all tests conducted within a borehole this simplification applies as the radius is small.

While it may be thought that Equation 6 is adequate to describe the situation in a test hole, it really needs some terms to allow for well bore loss. This describes the imperfect connection between the test hole and the ground surrounding it. The petroleum industry uses the term skin ( $S$ ) to describe this. In this paper we will use the symbol ( $S_k$ ) to differentiate it from storage terms used in hydrogeology. Incorporating this term into Equation 6 leads to Equation 7. Here the radius is the test hole (well) radius  $r_w$ .

$$p_{r_w,t} = p_i - \frac{q\mu}{4\pi kh} \left( \ln \frac{4kt}{\gamma\phi\mu cr_w^2} + 2S_k \right) \quad (7)$$

Equation 7 forms the basis of analysis in a single hole.

In the case of a constant flow from, or into a test well, the pressure (head) should change as a straight line with respect to the natural log of time with a slope,  $m$ , given in Equation 8.

$$m = \frac{q\mu}{4\pi kh} \quad (8)$$

Figure 1 shows the theoretical distribution of head around a well test zone that has been pumped for a period. The head decreases from the initial pressure towards the well bore and would intersect it at a certain value. The actual head within the well is however less due to the near well bore losses.

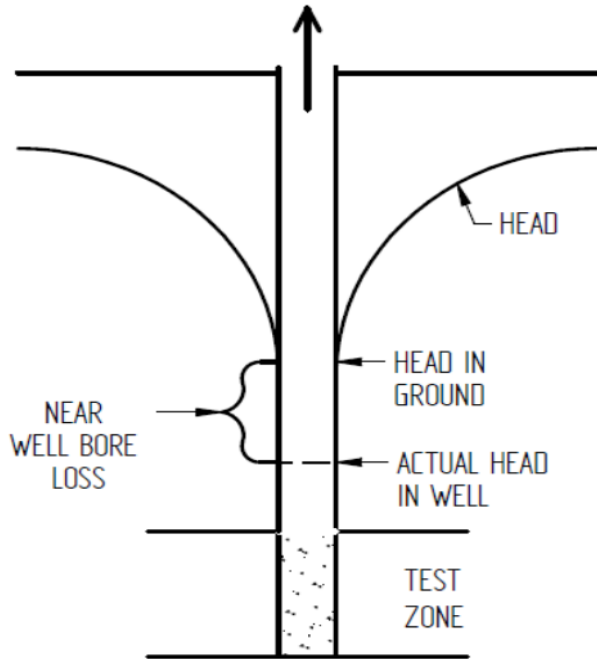


Figure 1. Theoretical potential head distribution around and in a pumped well.

It is also possible to use the principal of superposition to analyse for variable flow rates and, in particular, for a flow period followed by one where there is no flow. This can be accomplished by adding well responses to flow and negative flow periods to create a flow block. This latter case is remarkably important in testing because the time after flow represents a case where the skin, or well bore loss term, disappears. This is good because a varying skin term will distort the straight line plot at constant flow. Varying skin terms are a function of pressure in the borehole and progressive changes due to plugging or erosion of the hole wall.

Equation 9 gives the pressure in a well following a period of uniform outflow. This is an immensely useful equation as it shows that the permeability may be calculated from the flow rate and the slope of the plot of pressure versus a log function of time. It is independent of skin or storage terms. In petroleum terminology it is called a Horner build-up (Horner, 1951) plot while in groundwater terms it is called Thies' recovery method (Thies, 1936).

$$p_w = p_i - \frac{q\mu}{4\pi kh} \ln \left( \frac{T+\Delta t}{\Delta t} \right) \quad (9)$$

Where  $p_w$  is the pressure in the well  
 $p_i$  is the initial pressure  
 $q$  is the outflow rate  
 $T$  is the flowing time  
 $\Delta t$  is the time after flow ceases.

Superposition theory may be used to create a multi-flow rate form of Equation 9. This enables uneven flows to be analysed. Equations 4 to 9 may be found in the excellent text by Dake (1978).

The skin term may be related to an effective borehole radius by Equation 10.

$$r_e = r_w e^{-s_k} \quad (10)$$

Where  $r_e$  is the effective borehole radius

It is not possible to extract information on the storage behaviour of the ground nor the directional nature of permeability from a single test hole. These must be derived from tests where the pressure is monitored outside of the test hole by the use of piezometers. Pressures measured by external piezometers are free of skin effects.

Where measurements are made in piezometers that are at some distance from the test well, the value of Equation 5 will frequently exceed 0.01 and the full form of Equation 4 will describe the pressure change with time, as opposed to the form shown in Equation 6.

The theory and equations presented above refer to the transient behaviour associated with flow to or from a well that fully penetrates the test zone in a formation of infinite lateral extent. Where there are barriers to flow, such as a dyke or a recharge boundary, then these change the response.

Testing within sealed boundaries is covered very well by the semi-steady state solutions used by the petroleum industry (Dake, 1978). Essentially once the transient response has passed, the reduction in pressure or head declines proportionally to the volume of fluid withdrawn. Such analyses can be very useful in determining the behaviour of a basement enclosed by a diaphragm wall.

## 2 THE REALITY OF TESTING

In practical terms, testing is conducted over a section of a borehole. In rock, this section may be defined by placing packers to straddle the test zone. In soil, it is an open section of borehole that might typically have been drilled beyond a cased zone or where the hole has been drilled with casing and then the casing is pulled up by a casing section.

The question of what fluid the hole has been drilled with is critical. Typically, holes are drilled with a fluid that contains some viscosifier within the mud to assist the lifting of cuttings from the hole while drilling. Even in cases where no viscosifier is used, the fines created by drilling change the nature of the drilling fluid, especially where the ground contains clays.

This fluid tends to penetrate into fractures and pores of the ground, thus blocking the zone around the well bore while drilling. The situation gets worse when permeability measurement is attempted by injection, as any of this fluid and any suspended particles are driven into the pores and fractures. Testing by drawing fluid out of the ground surrounding the borehole is an inherently more reliable method of determining its permeability because the wellbore loss tends not to increase and the fluid drawn out is that which exists within the ground. The latter matters where clays may disassociate when subject to the different fluid chemistry of the injected fluid (water).

Most permeability tests conducted for geotechnical purposes are conducted in a single borehole. By definition, they cannot therefore provide information on directional permeability or storage behaviour of the ground. Unfortunately, very few of them are conducted in a manner that provides any useful information on the permeability nor of the pressure of fluid in the ground.

The best of the test processes in use are those that attempt to measure the transient behaviour of the well. If these come with pressure monitoring in adjacent piezometers, this is better still as

skin terms disappear. Also, anisotropy and storage parameters may be determined. Further, if a pumping test is conducted for long enough, it may show quite a lot of other features. These can include the effects of barrier boundaries, recharge boundaries and delayed yield. The latter being a phenomenon where an upper layer provides water to a lower one through some form of low permeability layer. To show such behaviour, tests need to be conducted over an extended period.

It is much more normal for most testing for geotechnical purposes to be conducted in a single hole. This can yield reasonable results for permeability and fluid pressure provided that proper procedures and analysis are used.

The correct procedures require stabilisation of fluid pressure prior to testing followed by a flow period during which both flow from or into the hole is measured, along with the pressure in the hole. Preferably, there is also a means to stop flow occurring, while the pressure continues to be measured. These procedures need to be followed by suitable analysis of the transient behaviour of the pressure in the borehole.

Unfortunately, it is common for none of these criteria to be met. The commercial pressure that exists to conduct measurement quickly leads to results that are generally worthless. Also, the reliance on some form of steady state behaviour that assumes that uniform ground conditions exist up to the borehole wall are hugely in error.

One of the worst examples of geotechnical testing used for permeability is the packer test which is generally conducted in rock. In this, either a straddle packer or a single packer and the hole bottom are used to delineate a test zone. Water is then pumped through the drill string into this test zone. The nominal pressure at surface for testing is 1.0 MPa and the flow is measured once some form of flow stabilisation is reached. The value of the test is reported in Lugeons after the inventor of the test (Lugeon 1933) and is defined as shown in Equation 11.

$$\text{Lugeon Value} = \frac{qP_o}{LP} \quad (11)$$

|       |                |                                                               |
|-------|----------------|---------------------------------------------------------------|
| Where | L              | is the test zone length                                       |
|       | P              | is the injection pressure at surface                          |
|       | P <sub>o</sub> | is the reference injection pressure<br>= 10 bar $\cong$ 1 MPa |
|       | q              | is the flow in litres/minute                                  |

This test was devised to determine how grout would be taken in foundations. Houlsby (1976) considered the test using five different test pressures maintained over a period of five minutes and described the results in terms of laminar flow, turbulent flow, dilation of joints, wash out of voids and void filling. The value of the Lugeon has then been interpreted quite incorrectly as corresponding to hydraulic conductivity. The test cannot provide this because of:

- The lack of initial pressure measurement hence the pressure difference driving flow.
- The assumption that steady state conditions exist and therefore no consideration of transient behaviour.
- A failure to consider near well bore loss behaviour.
- The problems associated with injection testing.

The test does provide a measure of water take at a given pressure under the conditions that exist during its execution, but that is all. It is an index test only. Such tests measure a response which is related to a number of parameters, but which are inseparable.

The slug test is used in various guises. They involve filling a hole up with water very quickly and measuring the rate at which it falls; alternatively, by withdrawing water very quickly and measuring the rate at which it fills up. The test is therefore one of variable flow and lacks any precise stop to the flow behaviour.

There are a number of approximate solutions to slug tests. Hvorslev (1951) assumes the flow rate is proportional to the head

difference between the well and the stabilised head before disturbance. Bower and Rice (1976), assume a form of head decline that does not follow the form of Equation 4. Cooper, Bredehoeft and Papadopoulos (1967) make alternative assumptions to the form of head decline which differ from Equation 4. Binkhorst and Robbins (1998) attempt to deal with drainage of a sandpack around a well screen in a hole. None of these deal with the effects of skin, though some attempts are made to deal with partially penetrating wells into the test formation. Stewart (2011) does present a solution to the slug test which includes skin. The solution is however dependent on the value of skin remaining constant. This is seldom the case where flow is into the borehole from the ground, and is never the case where flow into the ground from the hole occurs.

This is very serious because skin values may easily range from -5 to 30 and may contribute to most of the pressure drop between the pressure in the ground and that in the test hole. While the practice of developing a test well by surging or jetting, may reduce the skin value, it does not guarantee that the skin will remain constant through a test, especially if that test involves injection.

Maintaining a constant flow rate from or into a hole is difficult, especially where there is no idea as to what the response of the hole will be. If a full pumping test is being undertaken, then a short test may be made to find out what the likely behaviour is, and then the flow rate may be adjusted for the longer test. It is usual in most tests for the flow rate to vary during the test. This requires robust analytical solutions that take account of this. These solutions should also be immune from the effects of changing skin. This will be related to pressure and flowrate in the test hole and may also change due to plugging or erosion of the well bore.

The latter effects essentially rule out the use of analysis of well response during the flowing period of the test.

While storage change within the well bore is the source of flow in a slug test, it is frequently ignored in analysing a pumping test, and in lower permeability cases it needs to be taken into account.

If piezometers are used to monitor the test, then these need to measure pressure change free of the effects of changing head and storage. While the approach of monitoring an adjacent open well may be acceptable in a high permeability condition, this is not generally the case, and any form of piezometer should be sealed in place and have a pressure transducer with minimal volume change.

#### 4 A BETTER TEST APPROACH

The foregoing discussion of well test theory and the practical problems of testing require a test method where the flow is preferably from the ground. It should also account for a variable flow rate and should analytically eliminate the solution from the effects of skin.

Variable flow rate can be dealt with firstly by calculating the rate of fluid withdrawal or injection properly, taking account well bore storage. Secondly it requires a solution that deals with variable flow rates. This can be achieved by the superposition of individual flow rates.

The need to account for the effects of variable skin can be eliminated by using a piezometer adjacent to the test hole. Where only a single hole is being tested, the only way to eliminate the effects of variable skin from the analysis, is to stop all flow at the end of the test and to wait for pressure recovery. The analysis is then based upon the flows and the pressure recovery and not on the pressure changes during the flow period.

In rock this means having a packer test system where a test section is sealed and subject to flow. At the end of the flow period, a downhole valve is used to seal the test zone and the pressure within the test zone is monitored by an electronic pressure transducer coupled to a logging system. This should

preferably have an on surface graphical display to show the pressure measurement history with time and in its varying analytical forms.

The operation of such a tool is shown in Figure 2 and Figure 3.

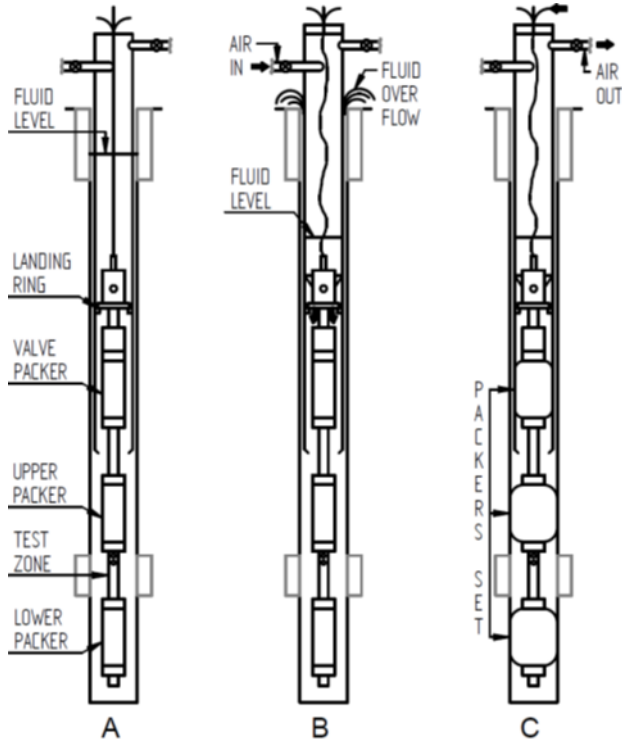


Figure 2. The operation of a Sibra drill stem test tool – first three stages.

The sequencing of operation is as follows:

- A. The wireline DST tool is lowered through the HQ drill string and a head seal is placed at the top of the drill pipe.
- B. The DST tool is shown landed and locked into the core barrel. Compressed air is used to push down the water level in the drill string. Overflow at the top of the hole occurs.
- C. The packers are inflated.
  - a. Compressed air is bled off.
  - b. The test zone is allowed to come to equilibrium.
- D. The valve is opened so inflow can take place.
- E. The valve is closed so that a head build up can take place.
- F. The packers are deflated and the tool can be pulled out of the hole.

This tool may also be used for injection, or more commonly a falling head test with shut in at the end of the flow period. The key to its success is the downhole valve, the surface read out and the ability for the system to withdraw fluid as opposed to injecting it.

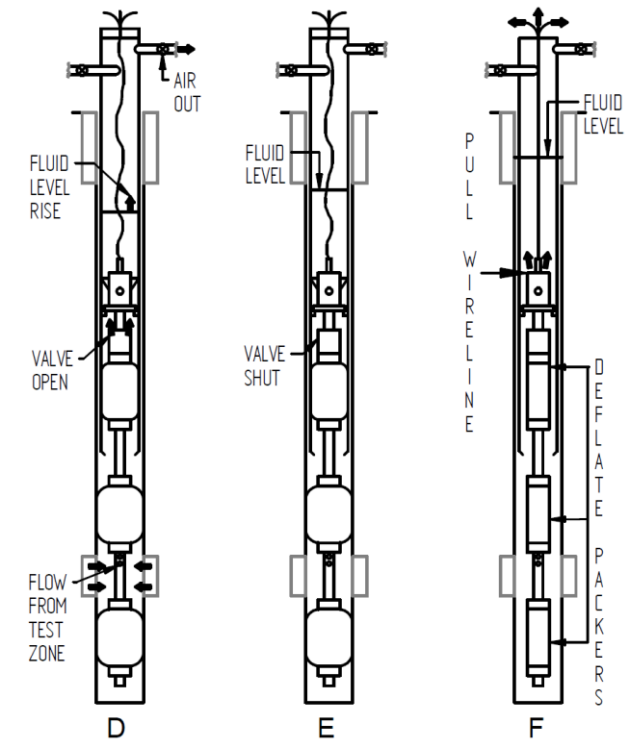


Figure 3. The operation of a Sibra drill stem test tool – last three stages.

An option for similar test for use in soil is shown in Figures 4 and 5.

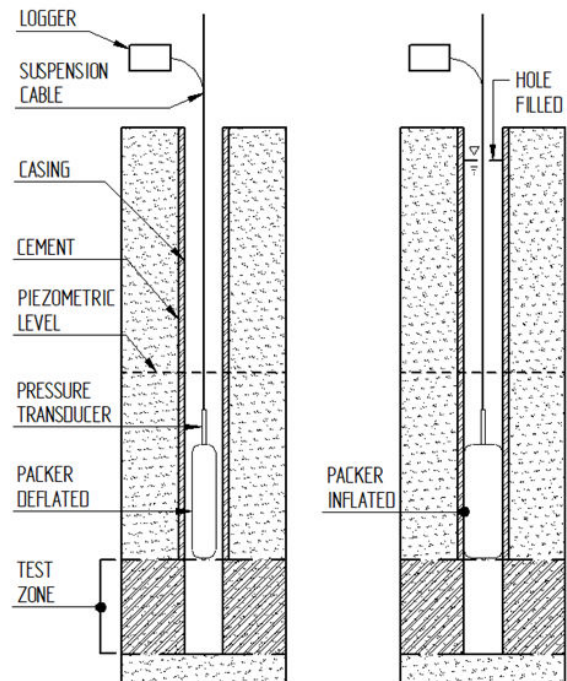


Figure 4. First two stages of a test in soil.

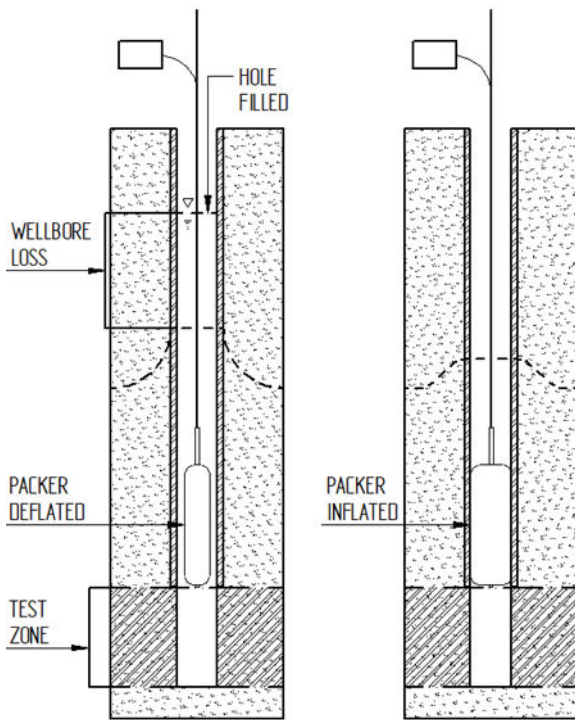


Figure 5. Second two stages of test in a soil.

In Figure 4 a casing is installed and cemented in a hole. This has been then drilled further to expose the test zone. A packer is lowered into the casing above the test zone and inflated. This packer is fitted with a pressure transducer that will monitor the test zone and ideally one that monitors the pressure in the casing above. The test zone is allowed to come to a stabilised pressure and the casing is filled with water.

In Figure 5 the packer is deflated and flow takes place from the casing and into the test zone. This flow can be deduced by the change in pressure. Before equilibrium head is reached, the packer is inflated to achieve shut in. An alternative to using the packer as a valve is to fit an in-line slide valve above the packer which operates in a similar manner to the drill stem test tool in rock.

The analysis of these tests requires the interpretation of graphs. Three plots are required. The first is a plot of the total test so that the operator and analyst can see what is going on. The second is that of a derivative plot of the pressure in the well after shut in with respect to a function of Agarwal time. The third is that of a plot of pressure versus the time function of Equation 9, or a more complex version if a multiple flow rate test is used.

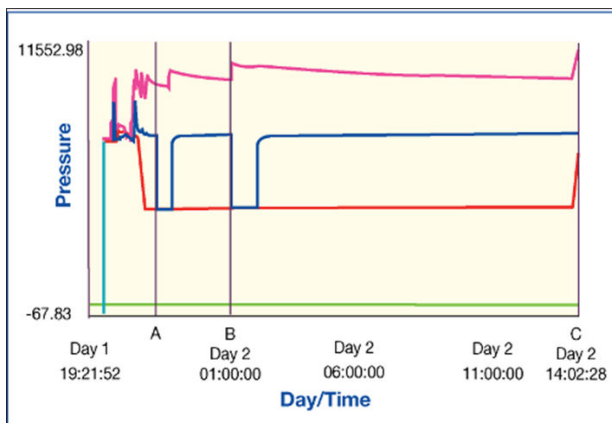


Figure 6. A total test plot of a drill stem test.

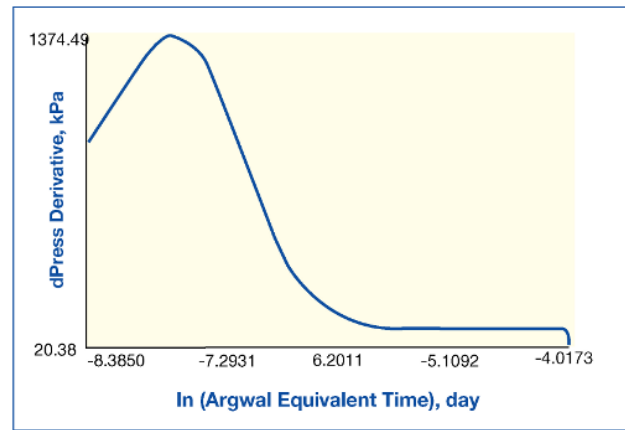


Figure 7. An example of a derivative plot.

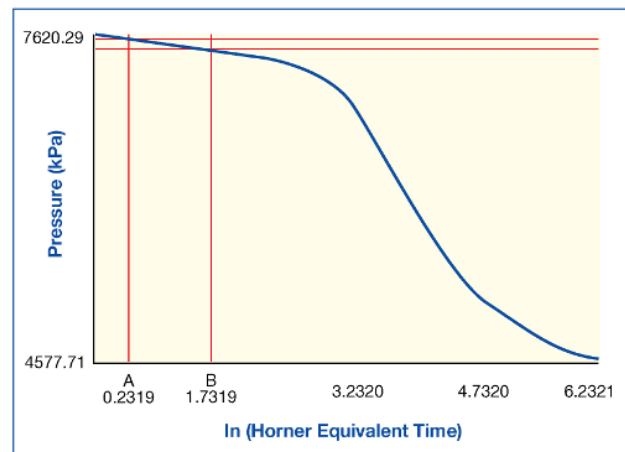


Figure 8. The build-up plot with respect to Horner Equivalent time.

In Figure 6 there are two flow periods followed by shut in and build-up. It is worth noting that very little inflow occurs when the valve is opened as can be seen by the nearly flat pressure change sections when the valve is opened. In fact some inflow has occurred. Following shut in after inflow, the pressure rise is almost instantaneous. This is the case because the skin is very high, and in the absence of flow through the well bore of the test zone, the pressure drop associated with it disappears. Following this there is the important recovery period used in analysis.

Figure 7 shows the derivative of pressure with respect to the log of Agarwal time (Agarwal, 1980). In this, time advances to the right and stabilized conditions on which to base analysis are shown in the flat line portion to the left of the plot. This time is used in the Horner plot shown in Figure 8. Here time advances to the left and the straight line portion of the plot provides the slope required for the determination of permeability from Equation 8. If a multiple flow rate test occurs, then the equations need to be modified by superposition to reflect this. The basics of a derivative plot and Horner plot remain. Extending the plot to the left to where the log of time becomes zero, corresponds to infinite time and enables the determination of the pressure in the ground.

It is important to note that the analytical portion of a transient test does not usually occur until some considerable time after shut in.

## 5 THE SIGNIFICANCE OF THE TEST

The question remains as to the significance of a test. This is linked to the volume or area of ground that is tested. In low permeability ground it is quite possible to conduct and analyse a test in which the volume of the ground in which the pressure

changes take place is so small that it has very little meaning, indeed it may lie within the effective radius of the well as defined by Equation 10. The author uses the concept of a mean effective radius of investigation to determine the extent of the effects of a test.

The mean effective radius of investigation is defined as being the radius calculated by:

- The calculated integral with respect to radius of the change in pressure over the analytical period used in well analysis divided by the pressure change at the well bore.

The mean effective radius must extend well beyond the effective well radius for the test to have any meaning.

Because the ground is frequently inhomogeneous, it is generally necessary to make multiple measurements. The ground may also be anisotropic. If a single well pumping test is undertaken with surrounding piezometers, then the test result is dominated by the behaviour around the well. If only three piezometers are monitored, it is theoretically possible to derive a perfect measurement of permeability including anisotropy. The only indication that inhomogeneity exists would be the varying permeability between that derived from analysing the response of the pump well compared to that of the average of that derived from the piezometers.

## 6 MULTIPLE PULSED TESTS

To overcome the problems described above, the author has used the procedure of conducting a drill stem test (DST) in one hole. This provides a measurement of permeability. If an estimate of the storage parameters is made ( $c\phi$  or storativity) then it is possible to estimate a suitable radius distance at which a second hole may be drilled and a test undertaken that will enable the pressure pulse of a second DST to be observed. If the first hole is fitted with a piezometer, the second hole is then drilled and tested, a second mean value of permeability may be obtained, along with the directional permeability between the two holes. This procedure may be extended to third and fourth holes to enable the determination of average permeability and its variation, directional permeability, storage behaviour and pressure. This is described in detail by Gray (2015).

## 7 CONCLUSIONS

The main conclusion of this work is that the permeability testing is conducted in the transient pressure range. It cannot be obtained from steady state analyses. Sometimes analysis of semi-steady state flow is useful in bounded areas.

Because of the importance of changing skin affecting the permeability around a well, the process of being able to end a flowing test with a period of no flow and pressure recovery enables the analysis for permeability to not be influenced by it. Many tests do not achieve this, notably variants of slug tests that are frequently used.

The packer test does not enable the measurement of permeability. It was developed for a different purpose. The variant of the drill stem test system described in the paper provides a good system for the measurement of permeability in rock. Its operation resembles that of an oilfield drill stem test and the analysis of test results is well developed. A simple variant for use in soils testing can also be useful.

Where the storage behaviour of the ground is required, it is necessary to have piezometers adjacent to the flowing test hole.

Because of the inhomogeneity of the ground, it is often beneficial to have multiple tests. If the procedure of pulsing from one hole to another is adopted, it can be used to determine both anisotropy and inhomogeneity.

If real results are to be gained, it is not possible to hurry up a permeability test. Firstly, it is necessary to wait until pressure stabilisation has taken place before testing. Secondly, it is necessary to wait for the transient response to become analysable. In low permeability ground this may be a long time.

Finally, it is necessary to ensure that the test zone extends far enough for the test to be meaningful.

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## 9 GROUNDWATER ANALOGUES OF EQUATIONS

$$h_{r,t} = h_i - \frac{q}{4\pi kb} \int_z^\infty \frac{e^{-x}}{x} dx \quad (4)$$

$$z = \frac{r^2 S_t}{4Kbt} \quad (5)$$

$$h_{r_w,t} = h_i - \frac{q}{4\pi kb} \left( \ln \frac{4Kbt}{\gamma S_t r_w^2} + 2S_k \right) \quad (7)$$

$$h_w = h_i - \frac{q}{4\pi kb} \ln \left( \frac{T+\Delta t}{\Delta t} \right) \quad (9)$$

|       |             |                                        |
|-------|-------------|----------------------------------------|
| Where | $b$         | is the test zone thickness             |
|       | $h_{r,t}$   | is the head at radius $r$ and time $t$ |
|       | $h_{r_w,t}$ | is the head at radius $r_w$ & time $t$ |
|       | $h_i$       | is the initial head                    |
|       | $K$         | is the hydraulic conductivity          |
|       | $q$         | is the flow rate                       |
|       | $r$         | is the radius                          |
|       | $r_w$       | is the well radius                     |
|       | $S_k$       | is the skin term                       |
|       | $S_t$       | is the storativity                     |
|       | $t$         | is the time since flow starts          |
|       | $T$         | is the total flow time                 |
|       | $\Delta t$  | is the time after flow ceases          |